HUMAN VALUES IN WATER, SANITATION AND HYGIENE EDUCATION: INTEGRATION OF HUMAN VALUES IN MATHEMATICS AND SCIENCES

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INSTITUTE OF WATER EDUCATION

SOCIETY FOR THE PRESERVATION OF WATER

Content

1. Circumambient Water (Mathematical Topic: *Percentage*)

It is a well-known fact that water is present on the earth and in the human body. Let's see how much water is present on the earth and in our body through numerical values.

Primary, Secondary Lesson Plan

We can find water everywhere. In fact, the earth is sometimes called a water planet; it is filled with 1,400,000,000 km^3 (=1.4× 10^9 km^3) of water and over two thirds of its surface is covered with water. The fresh water on the earth, however, is no more than 3 *%* of this 1,400,000,000 km^3 water. Moreover, most of the fresh water is ground water, water vapor, snow and glaciers, which is unusable water. Therefore mankind can use only 0.8 *%* of water on the earth.

Question 1 *Where can we find water on earth?*

We can find water as oceans, rivers, lakes, ponds, glaciers, ground water and in the air as water vapor.

Question 2 *How much water is on the earth?*

a. 1,400 km^3
b. 140,000 km^3
c. 14,000,000 km^3
d. 1,400,000,000 km^3

The earth has d. 1,400,000,000 km^3 of water.

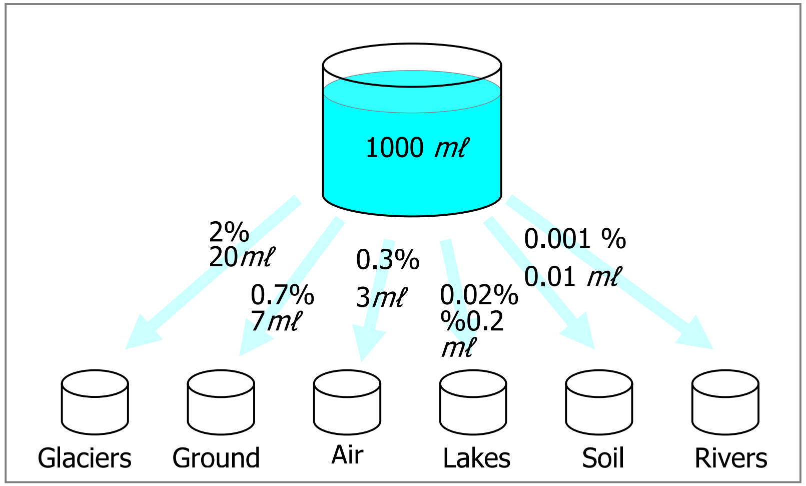
Question 3 *The following table shows percentages of water sources on the earth. Let's guess which are oceans, rivers, lakes, glaciers, ground water, vapor in the air and soil moisture.*

Sources of Water	Percentage %
①	97 %
②	2 %
③	0.7 %
④	0.3 %
⑤	0.02 %
⑥	0.001 %
⑦	0.001 %

The answer is

① Oceans, ② Glaciers, ③ Ground water, ④ The atmosphere, ⑤ Lakes, ⑥ Soil moisture, ⑦ Rivers.

Question 4.1 (Activity) Prepare a *1000 mℓ bottle of water and seven small beakers, and distribute 1000 mℓ water to each beakers according to the percentages in the above table, that is, pour 20 mℓ (2 % of 1000 mℓ) of water into the first beaker, 7 mℓ (0.7 % of 1000 mℓ) of water into the second beaker…. And label each bottle as "Oceans", "Glaciers (or Ice)", "Ground water", "Air", "Lakes", "Soil" and "Rivers".*



The big beaker expresses seawater and small beakers represent glacier, ground water, vapor in the atmosphere, lakes, soil moisture and rivers respectively. Children can identify the quantities of each source at a glance. Namely, most of the earth's water is sea water and the quantity of rivers water is very small.

Question 4.2 (Secondary Lesson (G10-12)) *Let's calculate each quantity exactly!*

This is just a practice of percentage, but children can appreciate how big the earth is.

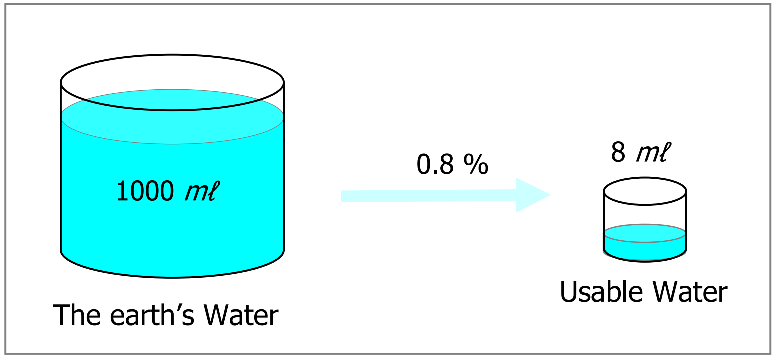
Sources of Water	Percentage %	Quantity
① Oceans	97 %	1,358,000,000 km^3
② Glaciers	2 %	28,000,000 km^3
③ Ground Water	0.7 %	9,800,000 km^3
④ Atmosphere	0.3 %	4,200,000 km^3
⑤ Lakes	0.02 %	280,000 km^3
⑥ Soil Moisture	0.001 %	14,000 km^3
⑦ Rivers	0.001 %	14,000 km^3
Total		1,400,000,000 km^3

Question 5.1 *What kind of water on earth can be used by us?*

From the above table, part of ② Glaciers, ③ Ground water, ④ Atmosphere, ⑤ Lakes, ⑥ Soil moisture, ⑦ Rivers are fresh water. However, glaciers, the atmosphere and soil moisture cannot be easily used by humans. Most people think that we have large quantity of water and that we can use it freely.

Question 5.2 *Useable water for human beings is just 0.8 % of the earth's water. Let us suppose that a 1000 mℓ bottle of water represent the earth's water. How much water in 1000 mℓ is usable for human beings?*

Using the above 1000 mℓ bottle of water as the earth's water, just 8 mℓ of water is usable for human beings.



The above questions have the following aims

- To calculate numerical values and percentages.
- To realize the preciousness of water, especially usable water.
- To appreciate the value of water and the earth.

Question 5.3 (Secondary Lesson (G10-12)) *Let us calculate how much water we can use!*

We do not often have to calculate large numbers. So this may be a good practice for children. Teachers can ask children what they think of the result of the calculation. For children who think the quantity is small, teacher can suggest how precious water is and for children who think the quantity is large, they can realize how large the earth is. The calculation for this question is as follows:

$$1400000000\ km^3 \times 0.8/100 = 11200000\ km^3$$
$$(= 1.12 \times 10^7\ km^3).$$

Question 6 (Activity) *Let's talk about water on the land of your school and country. How much area is covered with water?*

Over two thirds of the earth's surface is covered with water. Discuss the water area of your school and country, looking a map etc. If the school (country) has a large water area, we should thank nature for its good environment. If the school (country) has a small water area, we should be aware that fresh water is precious. The aims of the questions 6 are:

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・To realize the preciousness of water and to share the water with others.

・To be grateful for our environment and the earth which stores large amount of water.

Let's turn our eyes to a smaller planet, the body. The amount of water in the body is 60-70 % of the weight, and 80 % of a newborn baby's body is water. In our body, 90 % of the blood, 92 % of the retina, 80 % of the brain is water.

If we lose only 1 % of water in our body, we feel terribly dry, and if 2-4 % of water is lost, we get dehydrated.

Question 7 (Secondary Lesson (G10-12)) *How much water does your body have?*

Show how much water our body has by using glasses (bottles) of water after the calculation. The amount perhaps surprises children, and they should realize that water is very important for us. In case our weight is 50 *kg*, the amount of water is

$$50\,kg \times 60/100 = 30\,kg = 30\,\ell.$$

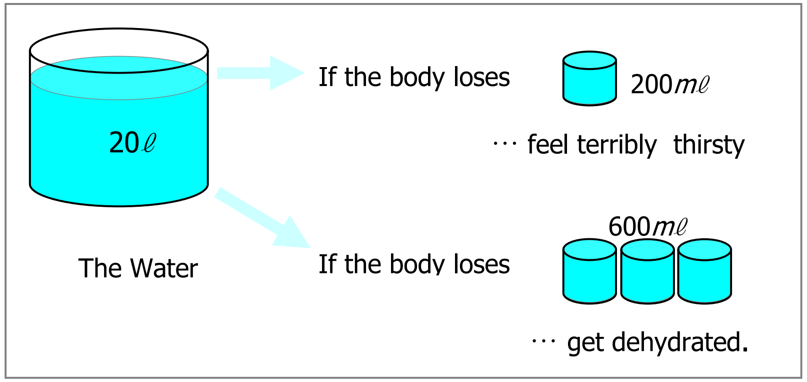
The aims of question 7 are:

・To calculate percentage and to understand the relation between weight and volume of water, that is, $1m\ell = 1g$ ($1\ell = 1kg$).

・To realize that water is an important part of our body.

・To be grateful for water and the body.

Question 8.1 (Activity) *Prepare a 20 ℓ bottle of water. This is the water in a body that weighs about 30 kg. Take 200 mℓ (1%) from the 20 ℓ of water. Children will feel thirsty when they lose water from their body.*



Next, take 600 mℓ (3 %) from the 20 ℓ of water. If children whose weight is 30 kg lose this 600 mℓ of water out of the body, they get dehydrated.

Question 8.2 (Secondary Lesson (G10-12)) *How much loss of water makes you feel thirsty and how much makes you dehydrated?*

In question 7, we took glasses (bottles) of water as much as our body has. Take 1% out of that water, which causes thirstiness. Next, take 3% of that water, which causes dehydration.

In case weight is 50 *kg*, the body has 30 *kg* (30ℓ) water, therefore

$$1\% : 30\,kg \times 1/100 = 300\ g = 300\ m\ell,$$

$$3\% : 30\,kg \times 3/100 = 900\ g = 900\ m\ell.$$

Namely, if a person, whose weight is 50 *kg*, loses 300 *m*ℓ of water in the body, the person feel terribly thirsty, and if 900 *m*ℓ of water is lost, the person is dehydrated.

Our body does not get dehydrated easily. We should realize how much our body takes care of us.

Human Values:

- **Be t*hankful for water that we receive***
- **Be grateful to our *earth for storing water for us***
- ***Respect our body***
- ***Learn to share water with every being***
- ***Love all, serve all***

We have to remember that water from the tap is also included in the 3% freshwater. This water is shared with all beings and plants. For this, with awareness that we are a member of the earth, love and compassion are essential for all human beings.

2. Activity on Circumambient Water

(Question 4 in Lesson 1, on Circumambient Water)

This is based on an actual class (Grade 11) at the Sathya Sai School on 19 January 2007.

Materials:

・1000 ml beaker.

・6 cups labeled as percentages, 2%, 0.7%, 0.3%, 0.02%, 0.01% and 0.001%.

・1000 ml tap water.

・Dropper or Pipette.

・Labels of "97%", "Ocean", "Glaciers", "Ground Water", "Atmosphere", "Lakes", "Soil" and "Rivers".

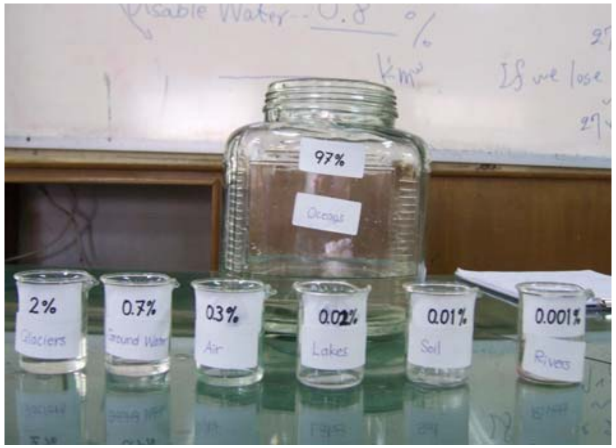
Procedure:

1. Fill up the 1000 ml beaker with tap water.
2. Pour in water into the cups according to their percentages.
3. Label the 1000ml beaker as ocean and 97%.
4. Guess which cup is “Glaciers”, “Ground Water”, “Atmosphere”, “Lakes”, “Soil” or “Rivers” according to their percentage, and label the 6 cups.
5. Check whether the guesses are right.
6. Discuss which water is useable for us?

Result:

We can infer that

- The percentage of water in the river is minimal.
- We don’t have usable water in large quantity, so we cannot waste water.



3. Activity: Drinking Water Project (Mathematical Topics: *Analysis*)

What is the role of mathematics? This may be a question which has been asked by many people a number of times. Analysis and estimate are important roles of mathematics as we know that mathematics is used in finance and economics to analyze the price of stocks, fluctuations in price and so on.

In the following activities, students will use mathematics to analyze and estimate the amount of water. These activities include statistics (graph, table), logical thinking for analysis and measures, presentation etc. Furthermore, it also can be expected that students realize the preciousness of water and cultivate the gratitude for water, which students are taught to think about this at the school and that they should take an interest in the statistics and compare it with their estimate.

In Project A, the activity was simple because the consumption was not so large and that the unit of the water is a bottle. In the Project B, we did a study of the consumption of water for the whole school which is vey important to understand.

Project A :

Primary, Secondary Lesson

It is said that in the year 2025, 40% of people will have a serious shortage of water in the world. Water is indispensable for us, but the quantity of water on the earth is about 1.4 billion cubic kilometer, that is a limited amount and not infinite. We must share this precious resource. First, let’s talk about solutions to this problem. The most practical solution is to use water only when necessary.

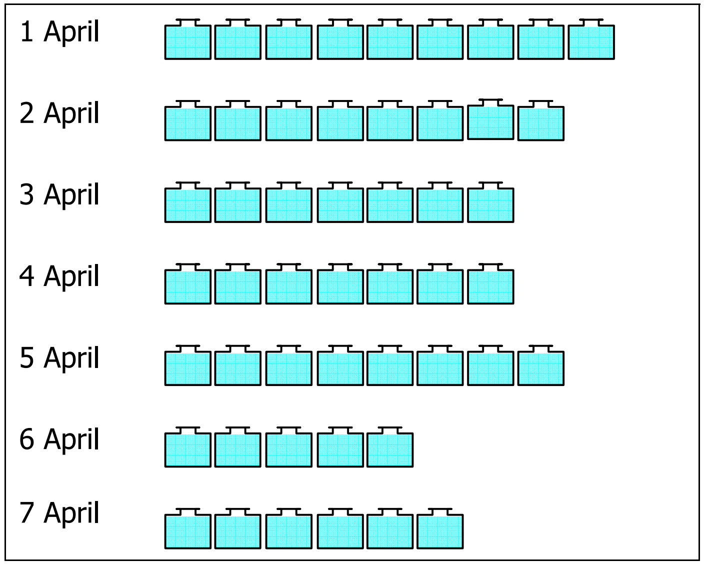
In many countries in the world, we must buy drinking water. Let's try to think of drinking water consumed in the school.

Step 1 : How much drinking water do we use?

First we must know how much drinking water is used up in a day/week at the school. Let's find out how much the consumption is and then make a graph of the result and present it. Then the teacher will ask the children what they think about their water consumption.

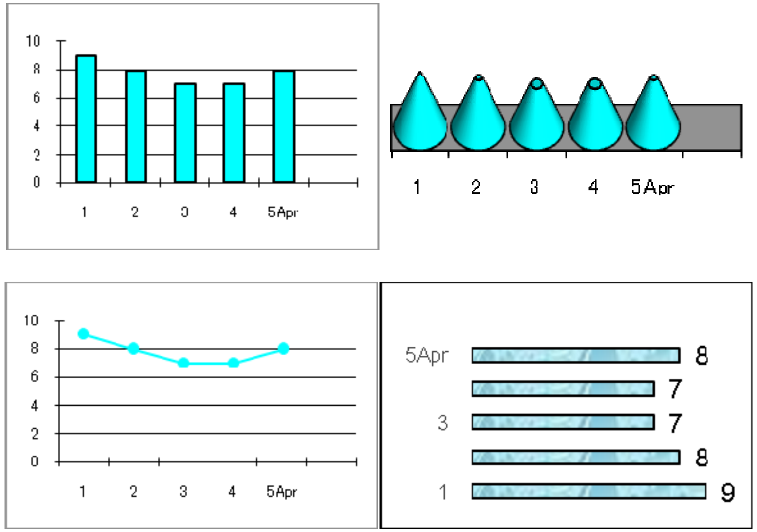
(Primary School Lesson for Step 1)

To look at the amount of water used, primary school children can count empty water bottles, and identify the number of water bottles that have been used up.



(Secondary Lesson for Step 1)

There are several ways to show the consumption, using bar graph, line graph, table and so on. Discuss which graph is most effective.



Step 2 : Analysis of the data

Let's discuss what are the connected issues regarding the consumption of drinking water. It may depend on the temperature, a schedule for the activities… Add these data to the former graph and observe it. Can we find a relationship between them?

Step 3 : How much water is necessary?

Estimate the consumption of drinking water from the results for next/each month.

About 2ℓ of water is excreted from our body in a day and it is restored by drinking water, food etc. We have to take about 1.2ℓ of drinking water in a day. How much drinking water does the school need at the minimum in a day? Water is essential for our life, but do we waste water?

Put the estimate on the graph made for the step 1 and compare it with the water used up. Can you make an accurate estimate of consumption?

Step 4 : Find an idea for reducing wastage of water and put it into practice

Let's talk how we can reduce wastage of water to a minimum and put it into practice. Will the idea work effectively?

Project B : ***How much water do we use?***

Secondary Lesson

Since water is indispensable for our life, the consumption of water in a day can be a very large quantity. We are fortunate to receive water as soon as we turn the taps on. It is therefore difficult to realize how much water we use in any given day.

Let us estimate how much water we need for showers, brushing our teeth, washing hands and face, dishwashing, laundry, toilet, cooking in a day. The following table is an estimated standard of consumption. Make our own table, if possible.

Consumption	**(ℓ)**
Shower	7
Brushing Teeth	3.8
Washing Hand and Face	3.8
Shaving	3.8
Dishwashing	20
Laundry	38

1. Compare your own consumption with the estimated data. What is the difference?
2. Where does the gap come from?
3. Brainstorming: How can we reduce excessive water consumption?
4. Carry out the ideas discussed and talk about the results.

Human Values: ***Grateful to nature for water, Sharing, Love & Compassion, Flexibility and Resourcefulness.***

The quantity of water on the earth has a limit. We should also cultivate love for the school as a member of the school by considering how we can reduce the consumption of water in the school.

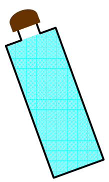
4. Infinite Series (Mathematical Topics: *Infinite Series*)

"Infinity" can be very mysterious. We know 'infinity' is found in nature. But if we are asked what 'infinity' is, it is not easy to answer. For mathematics, infinity is an important concept and it is an interesting topic to inquire. We can catch a glimpse of the beauty, diversity, perfection, inclusiveness of nature through infinity.

The following questions deal with finite quantity of water and infinite number of people (as total number). In the first question, students may have an interest to know that water is getting scarce, but water in the bottle never runs out. On the other hand, in the second question, the total amount becomes infinity. Although we usually use finite/infinite series, logarithm, integration etc to solve these kinds of questions, these two questions can be proved without them. So, the lower grades students can try them.

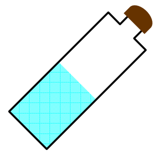
Primary, Secondary Lesson

A man was traveling across a barren desert with only one bottle of water, from which he had drunk all the water. It was very hot and there was no water in sight, only a mirage where he had hoped to find more water. As he struggled onwards his prayers were answered when he found a beautiful oasis.



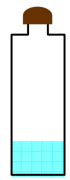
There he rested and happily drank water from the oasis, then filled his bottle with water and continued on his journey.

After some time he met another very thirsty traveler. He gave his bottle full of water to him. This very grateful traveler drank half of the water and continued on his journey.



He then encountered a third man who was nearly unconscious and dying of thirst. Out of compassion, he offered the bottle of water to this third man. This man then regained his consciousness after drinking half of the remaining water (1/4 of the bottle).

This third man also came across a fourth man who, was suffering from dehydration and had just fallen off his camel and was about to breathe his last breath. Again out of compassion he gave the bottle with the remaining water (1/8 of the bottle). The fourth man also drank just half of the remaining water.



The story continues with many people being helped by drinking half of the remaining water from the same bottle which each person received from the other.

Question 1 (a) *How many people could receive the bottle and drink half of the remaining amount of water?*

a. 3 people
b. 10 people
c. 100 people
d. uncountable people

The answer is d. Question 1 (a) can be changed to the following question 1 (b).

Question1 (b) *When will the water in the bottle run out?*

Secondary school students can solve this by using geometric progression. But this is also understood easily even by primary school students, if they notice that any man drinks half of the water given by the former, that is, the same quantity of water is left in the bottle for the next. This means that water never runs out, therefore uncountable people can get water from the bottle.

Secondary Lesson (G10-12) for the Question 1

We consider the consumption until the n^{th} man is $1/2 + 1/2^2 + 1/2^3 + \cdots + 1/2^{n-1}$ bottle of water, that is a geometric progression and for all $n>0$

$$1/2 + 1/2^2 + 1/2^3 + \ldots + 1/2^{n-1} = 1-1/2^{n-1} < 1.$$

Thus, the consumption cannot exhaust a bottle initially full of water, that is, the water in the bottle never runs out.

Remarks: In solving the question, it is necessary for teachers to indicate that the number of persons is an infinite number, whereas we deal with a finite quantity of water. It is also interesting to ask students the following before they go to question 1:

Where can we find infinity around us?

How can we prove their infinity?

These questions also help students to observe nature and make them face nature. Talking of mathematical meaning, of course, students learn about infinite series and infinity, moreover, students probably realize it is more difficult than expected to prove what is taken as a matter of course.

Human Values: ***Sharing ,love & compassion, caring, the value of water.***

If the story is changed as follows:

The n^{th} person drinks 1/(n+1) of the bottle.

What will be the situation?

Question 2 *If the n^{th} person drinks 1/(n+1) of thebottle, how many people can drink the water from the bottle?*

a. 3 people
b. 10 people
c. 100 people
d. uncountable people

The answer is a.

Primary School Lesson (G7-9) for the Question 2

Students just have to calculate. The calculation is not difficult, if they can calculate fractions or irrational numbers like

- the 1st man drinks 1/2 bottle of water and 1/2 bottle of water is left
- the 2nd man drinks 1/3 bottle of water and 1/2 -1/3 = 1/6 bottle of water is left
- the 4th man drinks 1/4 bottle of water and 1/6-1/4 bottle of water, but 1/6-1/4 <0.

This means that the 4th man has drunk up all the remaining water in the bottle.

Secondary School Lesson (G12) for the Question 2

In this case, the consumption up to the n^{th} man is 1/2 + 1/3 + 1/4 + … + 1/n, that is, we have to find n that satisfies 1/2 + 1/3 + 1/4 + … + 1/n > 1. This can be calculated one by one as above.

As n increases to infinity, the consumption cannot reach 1 in the question 1. It is proved as follows: for all n>2

$$1/2 + 1/3 + 1/4 + \dots + 1/n > \int_2^{n-1} \frac{1}{x} dx \to \infty \; (n \to \infty).$$

The main point of the question for high-school students is the consumption 1/2 + 1/3 + 1/4 + ……… This problem can be changed by using trees as an example.

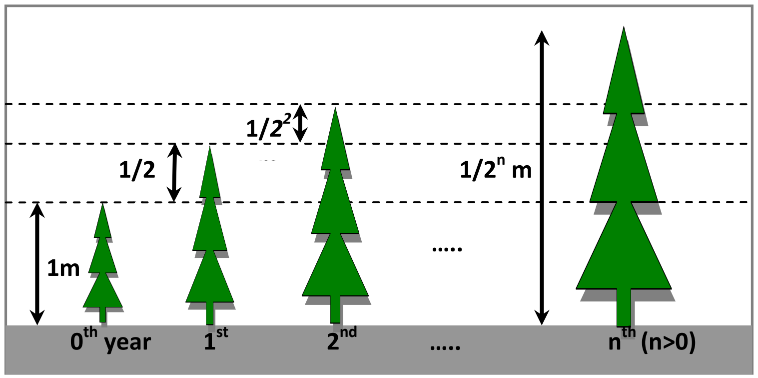
Students have a plan to grow trees at school. Because they had learnt that water and sunlight give life to plants, they watered the trees accordingly. Because of that, the trees grew rapidly to the height of 1m. The trees kept growing after that. They grew 1/2m in the first year, $1/2^2$m in the second year…, and in the n^{th} year they grew $1/2^n$m.

Question *What will be the height of the tree after a long time?*

The height approaches 2 m in due time, but it never exceeds 2 m. The height in n years is

$$1+1/2 + 1/2^2 + 1/2^3 + \dots + 1/2^n$$

and this question is solved in the same way as question 1.



5. Distance (Mathematical Topics: *Geometry*)

The following are practical questions and a familiar problem in geometry. For this problem, students can calculate theoretically on paper and can check it on actual location. We can also play a game by taking

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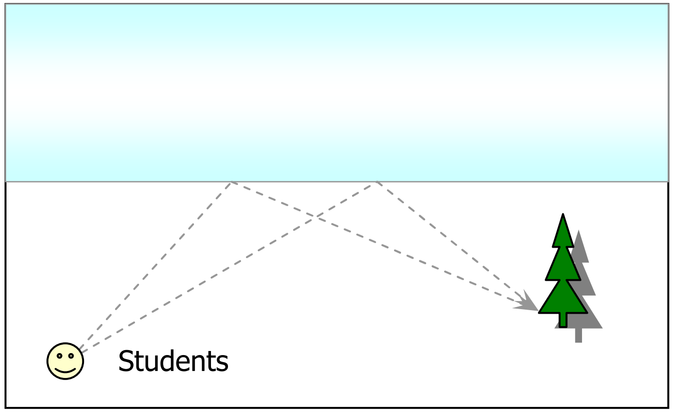
water from one place to another. The winner is the group that can move the most water. As part of the strategy for winning, students should calculate the shortest distance. In this activity, mathematics can be a strategic means. At the same time, students will bring out various values through group activity such as cooperation, unity, love, caring, calmness, peace, concentration, responsibility, etc.… Moreover, students have to carry water carefully so as not to waste the water, and that will help to bring about a more positive attitude towards water.

Primary & Secondary Lesson

On a hot day, students have to water trees at the school. There is a river near the school. They decide to take water from the river.



Question 1 *Students is going to water the trees by taking water from the river. At which point on the bank of the river must students take the water so that the total distance travelled is the shortest?*

Primary Lesson for Question 1

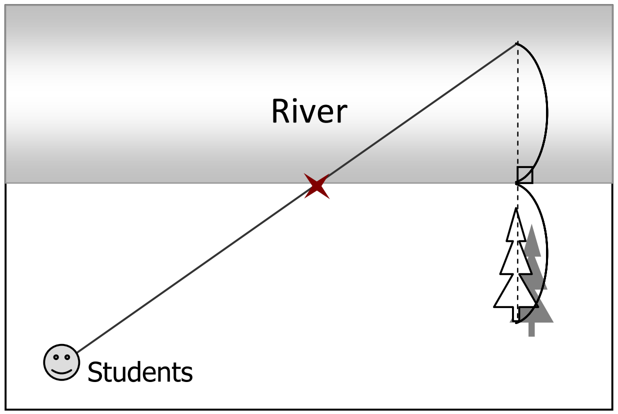
Students can measure actual distances of different routes and discover the distances are not the same. This may be enough for primary school students.

Question 2 *How can we prove that the route is the shortest?*

Secondary Lesson (G10-12) for Question 1

Secondary school students usually solve the problem in question 2 as follows:

First, draw a perpendicular line which is twice as long as the distance between the tree and the river. Then, draw the second line between the students and the top of the first line. The intersection point of the second line and the edge of the river is the answer. This lesson can be applied as an introduction to a lesson in geometry as a quiz.



Human Values: *Care for trees, Love, Cooperation, Unity, Flexibility, Curiosity*

Students should have many ideas through discussion in a group. Let students present their ideas. We can enumerate many valuable points derived from group discussion and presentation. For instance, the values of *cooperation*, *respect*, *love*, *caring*, *flexibility*, *forgiveness*, *patience* and *perseverance* are necessary to make friends. We should understand our own ideas as well as those of others. It is not important if their ideas are correct or not, because **mathematics does not have any useless idea. It is also an important result in mathematics to know which ideas do not work.**

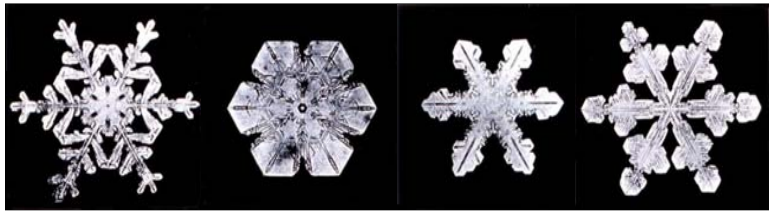
6. Snow Crystal (Mathematical Topics: *Hexagon, Polygon*)

Regular polygons had been studied to calculate the ratio of the circumference of a circle to its diameter, that is, if the number of side increases, the polygon approximates a circle. Polygon gives much material for several mathematical topics, for instance, square measure, triangles, angles and sequences.

It is well-known that the shape of snow crystals is a hexagon. It is said that the shape of snow crystals are never identical. They are very beautiful. We can also see the beauty of nature here.

Lesson

Water takes various forms on earth, and snow is also one state of water. A crystal form of snow is a beautiful regular hexagon.



Wilson Bentley, 1902

Primary Lesson

Question 1 *Where can we find a hexagon (n-polygon) around us?*

The above question has the following aims:

・To understand polygons.

・To get students to become interested in nature.

・To realize the beauty of nature.

We can find polygons in human designs such as a soccer ball, which consists of twelve pentagons and twenty hexagons, but it is not so easy to find them in nature. Let students try to find them in nature.

Let's talk about the properties of hexagon (polygons) by observing them, and make a hexagon on the basis of what are found in question 2.

Question 2 *What properties does a (regular) hexagon have?*

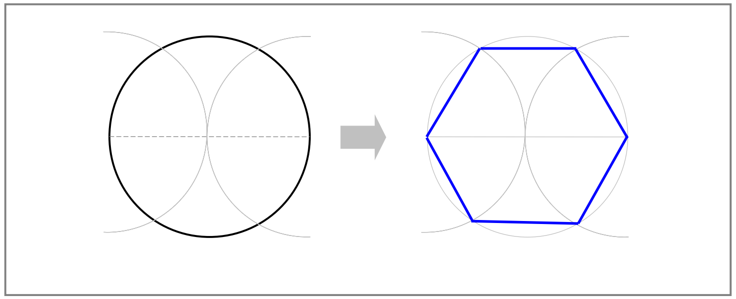
Generally, we can say that a regular polygon has all sides and all interior angles equal. But students can give any answer from their observation of the hexagon, for instance, *hexagon consists of several (how many?) isosceles triangles*, a *number of pairs of parallel sides* etc. It is important to grasp what hexagon is through observation.

Examples of Questions and Activities involving Hexagon

The following are examples of questions on hexagon (n-polygons).

Question 3 (Activity) *Let's make a snow crystal! How can we create a regular hexagon (with only a pair of compass and straight edge)?*

There are several ways to make a hexagon. It is good to present ideas they find in order to know that there is not only one way to get the same answer. Actually, we can draw a regular hexagon easily by using the fact that a radius is about one sixth of the length of the circumference.



Let's look at the hexagon we made. The following are examples of questions and activities about a hexagon:

・*What is the area and the length of the circumference of a hexagon?*

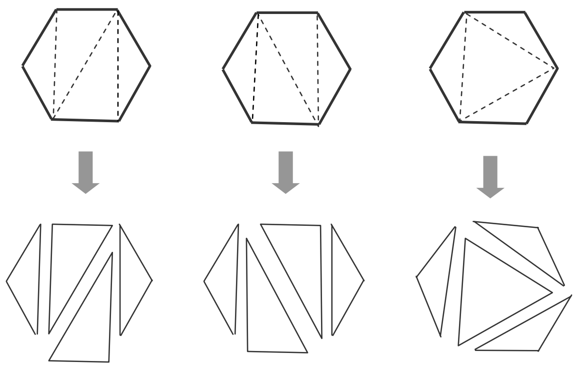
・*How many diagonals does a hexagon (n-polygon) have?*

How many diagonals do triangles, quadrilaterals, pentagons and heptagons have? Can we find a rule for the sequence of the number of diagonals?

* The number of diagonals of *n*-polygon is $n(n-3)/2$ (*n*>2).

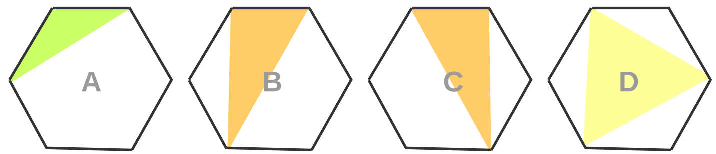
・*Make a puzzle by cutting up a regular hexagon into triangles and play with friends.*

Puzzle made from a hexagon



How many types of triangles does a regular hexagon have? What types of triangles are they?

Triangles in a regular hexagon



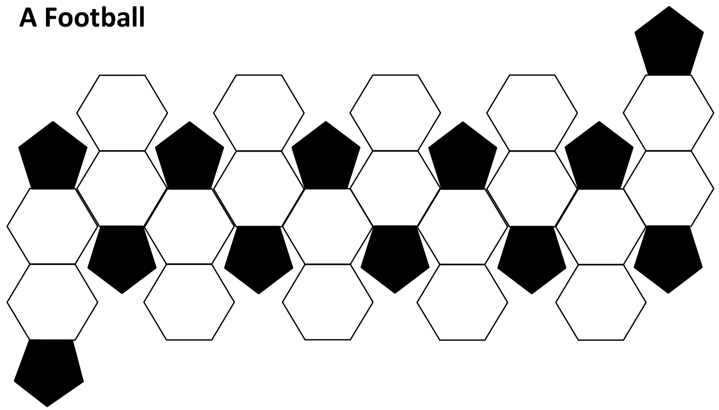
Isosceles triangle　Right triangle　Regular triangle

*The triangles B and C are opposite isometry.

・*Make a football with paper.*

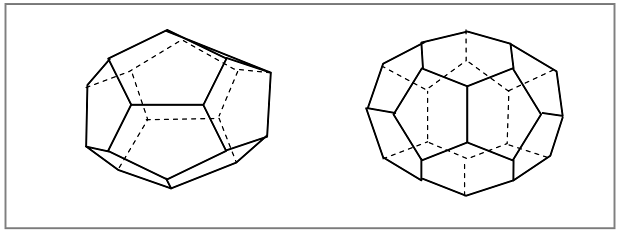
*A football consists of twelve regular pentagons and twenty regular hexagons.

A Football



As we know the above, twelve regular pentagons and twenty regular hexagons are arranged regularly to construct a football. Any piece cannot be missing. Misarranging the pieces, we will not be able to construct a proper football. Similarly, we all have our roles to play in society. We should not neglect our place of duty in cooperating with others. If so, what we are trying to carry out will not succeed.

It is interesting to note that the form of crystals of water can become many-sided shapes as follows according to different conditions.



Human Values : *Diversity, Flexibility, Resourcefulness, Unity, Curiosity, Uniqueness, Beauty of nature, Balance, Truth.*

We can find many mathematical related matters with a regular hexagon, which is the shape of snow crystals. Not only is mathematics latent in Nature but mathematics is a part of Nature.

In this lesson, especially in the elementary school lesson, students can learn about the properties of a regular hexagon (polygons) by carefully looking at a hexagon. Therefore, students can find different ways to solve problems with *flexibility*, *resourcefulness*, *curiosity* and so on. All ideas lead us to the same answer and the answer cannot be different. This means hat the answer is *truth*.

Each crystal form of snow has different pattern of regular hexagon. However, all crystal forms are beautiful. And Nature is beautiful. Similarly for humankind, not a single person is the same as anyone else. Each of us is *unique* and *beautiful*. *Balance* may be one factor for beauty. In fact, many architects and scientists have thought about beauty and what makes us feel beautiful. We can perhaps say that a regular polygon is beautiful. Let's think why we feel they are beautiful.

7. Cell & Unity (Mathematical Topics: *Exponential, Logarithm*)

It is an interesting point in mathematics to be able to touch huge numbers, which are sometimes beyond our imagination. The following problem concerns power and is an example of a small task that can result in a big solution. In mathematics, a system whose behavior is unpredictable by general methods because of its numerous or unknown components is called a complex system. And it is said that most biological systems are complex systems.

Secondary Lesson (G10-12)

Our body consists of 60×10^{12} cells, which hold 45 % of the water in our body. Thus, cell is a storeroom of water for our body. The number 60×10^{12} surpasses our imagination, and our body cannot exist without total cooperation/unity between cells. Let's consider how we can show a concept of cooperation/unity in mathematics.

If a cell cooperates completely with another cell, we say "the cell has 100% cooperation", that is, the degree of cooperation is defined by 1. If a cell has totally no cooperation with other cells, the degree of cooperation is defined by 0. Now we consider a case that all cells have 99.9% cooperation on an average *i.e.* the degree of cooperation is 0.999. We can say that all cells work very well together.

Question *What is the degree of cooperation of our body?*

a. 0.9
b. 0.5
c. 0.1
d. 0

To see it, we must compute $0.999^{60\times10^{12}}$. This is an exercise of exponent and logarithm as follows.

$$\log_{10} 0.999^{60\times10^{12}} = 60\times10^{12}\log_{10} 0.999 < -1.2\times10^{10}$$

where $\log_{10} 0.999 = 2\log_{10} 3 + \log_{10} 111 - 3$ and $\log_{10} 3 < 0.4772$, $\log_{10} 111 < 2.0454$.

Thus $0.999^{60\times10^{12}}$ is far less than $10^{-1.0\times10^{10}}$, thus, this value approximates 0. In fact, the value $\log_{10} 0.999$ in the above calculation is quite larger than its exact value, because we use the approximate values of $\log_{10} 3$ and $\log_{10} 111$. But it is still enough to know $0.999^{60\times10^{12}}$ is extremely small.

Human Values : ***Cooperation, Unity, Respect for our body, Love & Compassion.***

Through this lesson, we show, by using numbers, that small changes can cause damage to the whole system. In this case, each cell has 99.9% cooperation and that seems to be almost perfect cooperation for us, humankind, however, if each cell loafs on its duty slightly, then our body would lose the value of cooperation and its functions completely. We have to respect our body.

Remarks One aim of this lesson is to see cooperation visually. We can also apply it to a class and school by using the number of students in a class and school instead of the number of cells, 60×10^{12}. If a class has 50 students, the calculation is 0.999^{50} and in a school with 1000 students, the calculation is 0.999^{1000}. It is difficult, however, to calculate exponent by using logarithm, if we deal with a small number like 50 and 1000 for the number of students. For simplicity, we can take 0.99 or 0.9 instead of 0.999, although the feeling of perfection is reduced. The exponents for each case are:

$$0.999^{50} = 0.9512, \quad 0.99^{50} = 0.6050,$$
$$0.9^{50} = 0.005154,$$

$$0.999^{1000} = 0.3677, \quad 0.99^{1000} = 4.317 \times 10^{-5},$$
$$0.9^{1000} = 1.748 \times 10^{-46}.$$

The logarithms for each case are:

$$\log_{10} 0.999 = -0.0004345,$$

$$\log_{10} 0.99 = -0.0043648,$$

$$\log_{10} 0.9 = -0.0457575.$$

Teachers can show students these values or a graph of these logarithms for the calculation as an exercise.

8. Volume (Mathematical Topics: *Volume*)

Volume is a unit to measure the quantity of water. We also use water to find the volume of various objects. The following is a well-known episode in the life of the famous scientist, Archimedes (B.C. 287?-212):

Long ago there was a king and he had a pure gold crown, but there was a rumor that the crown was not 100% pure gold. It was said that the craftsman who made the crown stole some gold and replaced it with silver. The king asked Archimedes to verify the rumor. On the king's request, Archimedes was so worried about how to verify this that he forgot to eat, sleep,

take a bath…. One day, he happened to pass by a public bath house and he decided to go inside. He was watching water running over the bathtub as he soaked himself in the bathtub. Then it dawned upon him that the quantity of the water running over is equal to the volume of his body in the bath. Finally he realized how to find out the volume of the gold crown. We can see whether the crown is made of pure gold by its volume. As soon as he realized that, "I found! I found!" he shouted as he rushed out of the bath house.

Even if we make a box out of the same paper, the quantity is not the same. If students have time, they should make various boxes and compare the actual volume.

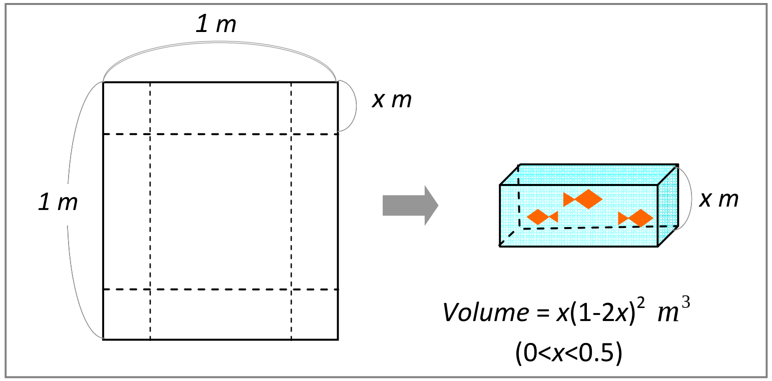
Lesson

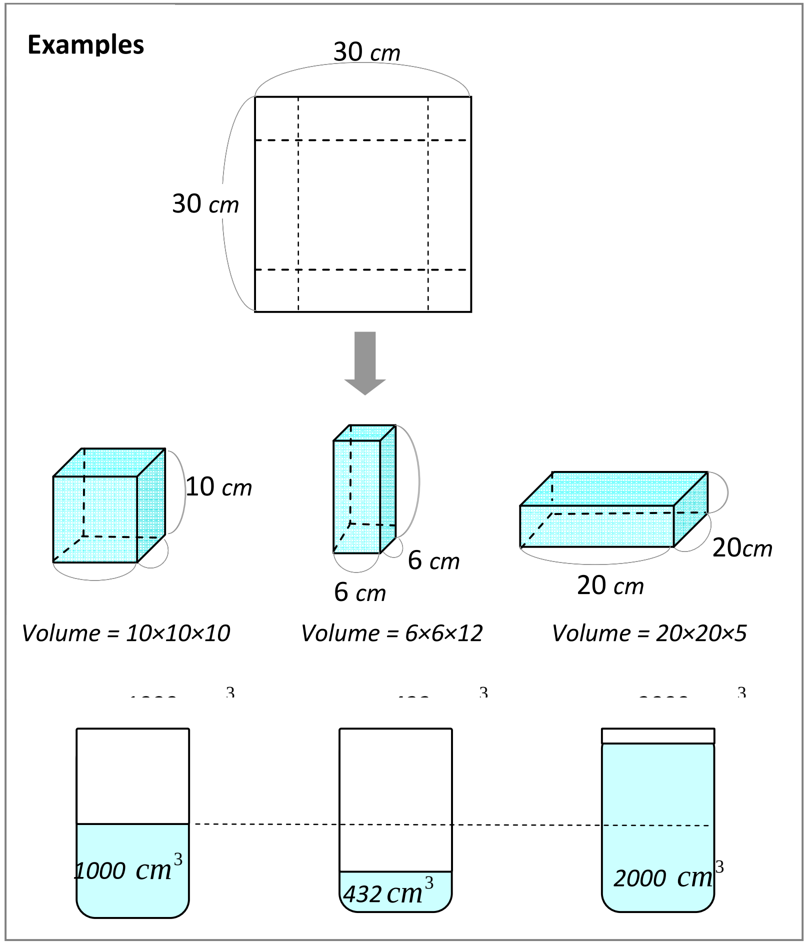
One day, students found a lot of fish in a pond, but the water was not clean. So the students decided to keep the fish in a classroom until they grow up. They now need to make a fish tank. The students bought a 1 square meter glass sheet to make a fish tank. The students think the bigger the tank is, the more comfortable it should be for the fish.

Question *Which box made out of the 1m square glass sheet has the largest volume?*

Primary Lesson

Let's try to make various boxes of paper of the same size and compare their volume in order to know the volume is not constant even if the boxes are made of the same paper. We can take any size of square paper and use sand as a substitute for water for measuring their volume. To measure the differences in the volume, teachers can get the students to use clear plastic cups to put the sand in.





Secondary Lesson (G12)

First, calculate volume of some examples similar to the ones shown above. And then let's consider the question generally as follows.

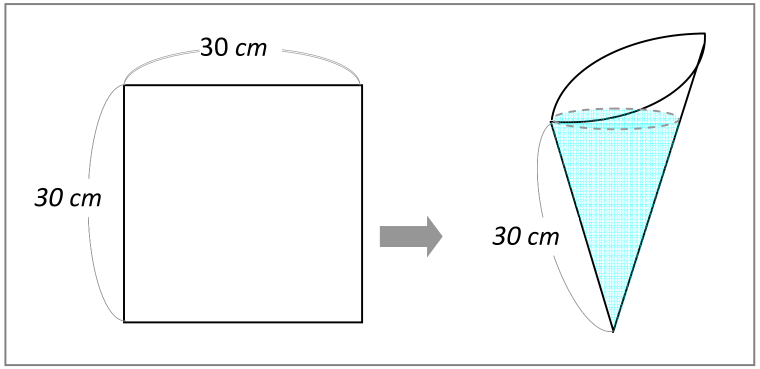
We want to find the value of *x* which gives the largest value of $x(1-2x)^2$. By differentiation, the answer is $x=1/6$ *m* and the volume is $2/27\,m^3$ (about $0.074\,m^3$). Students also can make a graph of volume to see its change when x changes.

We can change the situation so we can deal with other solid figures in addition to boxes:

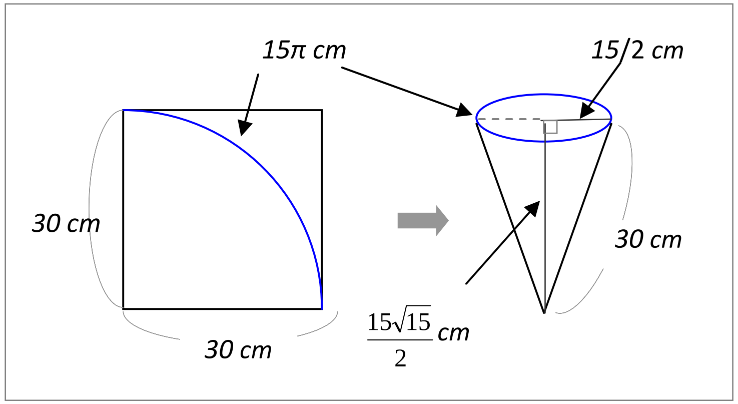
> *One day in summer, students were playing sports and then they became very thirsty. There was a big tank filled with drinking water, however, there were no cups. So, the students used a square piece of paper as cups and shared it with their friends.*

Question *How do we use a piece of 30 square cm paper for making a cup so as to get as much water as possible?*

In this case, the following conic cup is also included in addition to boxes.



The volume of the cone is $1125\sqrt{15}\,\pi/8\ cm^3$ (about $1711\,cm^3$). The calculation is



Then the volume is $\frac{1}{3} \times (15/2)^2 \pi \times \frac{15\sqrt{15}}{2}\ m^3$.

Remark We can also use rectangle paper instead of square paper. But the nature of the questions is not different.

Human Values : *Diversity, Flexibility, Modesty, Resourcefulness, Curiosity.*

The main point of this lesson is *diversity*. Of course, we know that we can make many products from milk, oil, wheat, gold, iron, plastic… Talking of paper, we just cut, stick, bend and fold. But things made of paper, even boxes, have different volumes, to say nothing of shapes. And water can be put in any shape. That is why any part of our body can hold water. We have to thank water for its modesty and flexibility. If students do this lesson as a group activity, we have more values, *cooperation, unity, respectfulness* and so on. We also put values such as *love, kindness, caring, peace, sharing* into the question.

9. Activity on Volume 1

(Question in Lesson 8, on Volume)

19 January 2007

Grade 11

Materials:

- 30 cm square paper.
- Ruler.
- Scissors.
- Tape.

Procedure:

1. Divide into groups.
2. Discuss how to make the biggest box from 30 cm square paper.
3. Make a box. Calculate the volume of the box.
4. Present the box and its volume.
5. The group that can make a box with the biggest volume is the winner.

Result:

We realize that

- Making boxes from the same paper, but the shape and volume are different.

10. Activity on Volume 2

Primary Lesson

Water is flexible. We can pour water into any shape. Our body also has water in large quantity. Not only human body, but also animals, plants, trees etc have water. This is because of the flexibility of water. Using water, we can measure volume.

Question *Let's measure the volume of various containers by pouring water into them!*

Human Values : *Flexibility.*

Water can be poured into any shape, but the nature of the water does not change. We are put in various situations. We should be flexible and gallant just like water.

11. Reflection (Mathematical Topics: *Transformation Geometry*)

Sathya Sai School, Thailand

Most things around us have shapes, and in them congruence and similarity are found. Some are found in nature, some are found in man-made things. For instance, petals and leaves of a plant seem to be congruent, a honeycomb has many congruent and similar hexagons, windows of a building are congruent, we can find similarity in the screen of a digital camera etc. We can construct a transformation between congruent or similar things. Also here we can find mathematics which is latent in nature. Water gives us also light and scenery by reflection and reflection is a transformation, too.

Sathya Sai School, Thailand

Primary, Secondary Lesson

A transformation in which the image is congruent to the object is called an **isometry**, and especially, if the image is a mirror image, we call it an **opposite isometry**. A transformation in which the image is the same shape as the object is called a **similarity**.

Question 1 *Where can we find isometry and similarity around us?*

The above question has the following aims:

・ To understand isometry and similarity.

・ To get students to observe our surrounding.

・ To realize the regularity of nature.

If the weather is fine, we can see reflection clearly on the surface of a pool, a pond or a lake, and the sky is reflected on glass covered building.

Question 2 *What type of transformation are these reflections?*

These are opposite isometries as in the case of a mirror. Ninety two percent of the retina is water, that means that we see by the reflection of objects in the water of the retina. But our feelings and reactions are not the same even if we see the same thing at the same time. How does that happen? What is inside us is reflected in our feeling, so this explain why we have different feeling. If the surface of the water is not calm, the reflection is also not clear. Within us, the mind should be calm in order to have a good reflection. At the same time, to have a good reflection, our environment is also important.

The surface of a lake often changes---it sometimes has ripples caused by a gentle breeze, it sometimes glistens in the sunshine and moonlight, at another time it reflects the scenery around. A great Impressionist painter, Claude Monet (1840-1926), was interested in the glitters on the surface of the water, and he seeks continuously to express the glistening of light on the surface of a lake throughout his life. However, the water is the same, even if its surface is different. Water is never self-assertive in spite of its importance. We want to learn its modesty, calmness, peacefulness, flexibility.

Human Values : ***Beauty and Regularity of nature, Changelessness, Thankfulness for water, Modesty, Calmness, Peacefulness, Flexibility.***

12. Virtual Water (Mathematical Topics: *Arithmetic*)

The quantity of the water on the earth has a limit. The consumption of water goes on increasing as the population increases and technology advances. More than one hundred million people on the earth still don't have enough drinking water in spite of the advancement of our civilization. We have to share water and we cannot waste water at all. Then, how much water do we use a day? It is said the world average consumption of water per person is 165 ℓ (FAO Aquastat 2003, NPO Waterscape). But, in fact, the average consumption for many countries, especially the developed countries, are well over the world average. We use a lot of water everyday not only for drinking, but also for the production of food. For instance, 22ℓ of water is used to produce 1ℓ of orange juice. The water used in the production process is called *virtual water*.

Countries that do not have enough resources must import many products from other countries. The quantity of water used in the production process is beyond our expectation. When we think about that, we realize that nothing can exist without water.

Secondary Lesson (G10-12)

First, let's discuss the following questions.

Question 1 *During the process of the production of milk, how much water is used?*

Question 2 *Guess how much water is needed to produce 1ℓ of milk?*

Let's see how much water is needed to make

one cup of coffee … 140 ℓ of water

1 ℓ of milk … 800 ℓ of water

1 *kg* of corn … 900 ℓ of water

1 *kg* of soybean … 2,500 ℓ of water

1 *kg* of wheat … 1,100 ℓ of water

1 *kg* of rice … 2,500 ℓ of water

100 *g* of beef … 2,000 ℓ of water.

The quantity of virtual water may differ from one country to another as well as types of product, but we should realize that we need a lot more water in order to produce them.

Question 3 *How much water is needed to make rice that we eat every day?*

If we eat 200 *g* of rice, 2,500×0.2 + (water used for boiling the rice) ℓ of water is necessary.

Even if the consumption of water in our country is not so much compared with the world average, other countries may use much more virtual water. For example, Thailand, the United States, Canada, Australia and Argentina are export countries of virtual water, and Japan, Korea, Sri Lanka, Italy and Holland import virtual water in large quantities. (Chapagain and Hoekstra, 2004 Water Footprints of Nations; UN/WWAP 2006, UN World Water Development Report 2) We have to remember that it is in helping one another that makes the world go round.

Through the study of virtual water, we can understand that a large quantity of water is being used.

Human Values: ***Sharing, Love, Cooperation.***

13. Pollution (Mathematical Topics: *Ratio, Tables, Graphs*)

Water is indispensable for us. But, using water also means that there will be waste or dirty water going down the drain. When we cook food and wash dishes, we drain away waste water. They cause pollution of rivers, lakes, and the sea. We can see the extent of pollution of a river by the BOD (Biochemical Oxygen Demand). The higher the value of BOD is, the heavier the pollution of the water becomes.

Lesson

Where does used water go? Water is indispensable not only for us, but also for all living things. We drink milk. Milk is good for us, however, fish cannot live in the milk. If we drain 200 *m*ℓ of milk, we need 3,600 ℓ of water to dilute the milk so that fish can live in it.**5**1

	BOD
Milk	120,000 *mg*/ℓ
Juice	110,000 *mg*/ℓ
Soy sauce	150,000 *mg*/ℓ
Soup	31,000 *mg*/ℓ
The water in which rice has been washed	900 *mg*/ℓ

It is easy to see the extent of pollution, using BOD. Fish can live in water whose concentration of BOD is 5 mg/ℓ maximum. The concentration of BOD in various liquid is as follows.

Primary Lesson

If we pour 200 $m\ell$ of the following food into the drain, we need to mix the following amount of water to enable fish to live in the solution.

Question 1 *Can a fish live in milk?*

Of course, the answer is "No".

Question 2 *If you pour a cup of milk (200 $m\ell$) down the drain, how much more water is needed for fish to survive in it?*

4,800 ℓ of water is necessary from the graph. Let's discuss with the students how big is the amount of 4,800 ℓ of water!

Secondary Lesson (G10-12)

If we drain away the amount (as given in Question 3) of milk, juice, soy sauce, soup and the water in which rice has been washed, a very big amount of water is necessary to obtain water whose concentration of BOD is 5 mg/ℓ.

Question 3 *How much water is necessary to reduce the water concentration of BOD to 5 mg/ℓ, if we drain away the following amount of milk, juice, soy sauce, soup and the water in which rice has been washed?*

	Necessary amount of water for a fish to live
Milk (200 $m\ell$)	① ℓ
Juice (200 $m\ell$)	② ℓ
Soy sauce (15 $m\ell$)	③ ℓ
Soup (200 $m\ell$)	④ ℓ
The water in which rice has been washed (2 ℓ)	⑤ ℓ

The answers are -

① 4800 ℓ.

The concentration of BOD of milk is 120,000 mg/ℓ, so 200 $m\ell$ of milk has 120000×200/1000=24000 mg of BOD. Putting x ℓ the amount of necessary water, the concentration of BOD is

24000/(x+0.2) = 5, and then x = 4800-0.2,

where the 0.2 in the equation is the amount of milk, 200 $m\ell$. Then about 4800 ℓ of water is necessary.

② 4500 ℓ.

It can be calculated in the same way as ①, where the concentration of BOD of juice is 110,000 mg/ℓ.

③ 450 ℓ.

The concentration of BOD of soy sauce is 150,000 mg/ℓ, so 15 $m\ell$ of soy sauce has 150000×15/1000=2250 mg of BOD. Putting x ℓ the amount of necessary water, the concentration of BOD is

2250/(x+0.015) = 5, and then x = 450-0.015,

where the 0.015 in the equation is the amount of soy sauce, 15 $m\ell$. Then about 450 ℓ of water is necessary.

④ 1200 ℓ.

It can be calculated in the same way as ①, where the concentration of BOD of soup is 31,000 mg/ℓ.

⑤ 360 ℓ.

The BOD of the water in which rice has been washed is 900 mg/ℓ, so 2 ℓ of that water has 900×2000/1000=1800 mg of BOD. Putting x ℓ the amount of necessary water, the concentration of BOD is

1800/(x+2) = 5, and then x = 360-2,

where the 2 in the equation is the amount of the water in which rice has been washed, 2 ℓ. Then about 360 ℓ of water is necessary.

Question 4 *How much water is needed for a fish to live in if you leave 2 mℓ of soy sauce on a plate after eating and pour it to a drain?*

60 ℓ of water is needed, because 6,000 ℓ of water is needed for 200 *m*ℓ of soy sauce.

Question 5 *The school has 300 students. All students leave a teaspoon (5 mℓ) of soup and they wash it away when they wash a plate after a meal. How much water is necessary?*

There are 300 students and each leave 5 *m*ℓ of soup, therefore, 5 × 300 = 1500 *m*ℓ of soup are left and drained. Because 1,200 ℓ of water is needed for 200 *m*ℓ of soup,

$$1200 \times 1500/200 = 9000\ \ell,$$

that is, 9,000 ℓ of water is necessary.

Human Values : *Sharing, Love & Compassion.*

We, human beings, are also a part of the ecosystem of life. If we drain away waste and fish should die out as a result, then animals that live on fish will also die, insects and planktons that fish feeds on will increase in number,…, and soon it will have an effect on us. The amount of what is left on a plate after eating may be small, but the total amount of water becomes sometimes huge. In fact, it is said that if each of us normally drains 250 ℓ of wastes a day and if each of us help to reduce the amount of waste, the total amount of water required to reduce the concentration of BOD would considerably decrease.

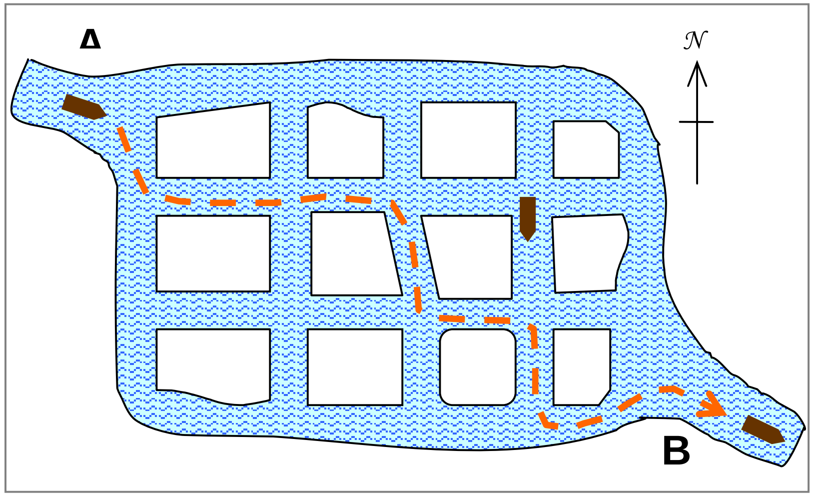
14. Transportation (Mathematical Topics: *Combinations*)

Water is not only for drinking, but also for transportation. In Thailand, canals used to be used as roads, and even now they are very important for boats used by travelers as well as traders carrying goods.

Secondary Lesson (G10-12)

Oceans, rivers and canals have a role in transportation. Let's consider the following!

There is a town which has fine canals as shown below.

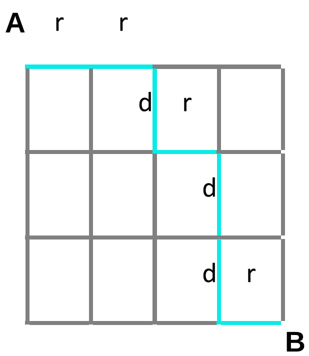


Everyday many boats come and go carrying a lot of cargoes, food, wood, steel, and other goods as well as people. People like to look at boats passing in the canals. People living in this town know the canal brings not only water and goods, but also peace of mind. The canal is indispensable for the inhabitants. And in the canals, boats give way to one another, so the canals are always peaceful.

Question 1 *How many routes are there between points A and B? The rule for this question is that a boat cannot go northwards or westwards.*

In this question, we try to find out how many ways are there between A and B with the shortest distance. The shortest way means that we cannot go up nor go to the left.

Putting "d" represent going downwards and "r" going to the right, distance from A to B can be expressed by a combination of three "d" and four "r" like "rrdrddr". This "rrdrddr" represents the blue route in the following figure.



Therefore, it is sufficient to consider how many combinations of three "d" and four "r" there are. Then the number of combinations of three "d" and four "r" is

$$ {}_7C_3 = \frac{7!}{3!4!} = 35. $$

Human Values : ***Love & Compassion, Diversity.***

Even such a simple case as the above has many routes. So in our life, if we have a goal to reach, there are many ways to achieve this. Each person has his own path, and not all are the shortest way. We cannot say which way is the best. What is important is to do our best and that we are taking steps towards the goal.